

# MATURA

## Mathematics

Spring Session 2011

### BASIC LEVEL

1. Given that  $f(x) = -3x + 5$ , find  $f(-\frac{1}{2})$ . For which  $x$  the value of  $f(x)$  is  $\frac{11}{2}$ ? Find the values of  $x$  for which  $f(x)$  is negative.

[8 points]

2. The first term of the arithmetic sequence is  $-4$ , the fifth term of that sequence is  $8$ . Find the common difference and the 100th term of the sequence.

[5 points]

3. Simplify.

$$((-a)^4)^3 \cdot (-a)^{-3} : a^9.$$

[5 points]

4. An acute triangle has the side  $b$  longer than the side  $a$ . The side  $a$  is  $\sqrt{17}$  cm long, the height to the side  $c$  is  $4$  cm long and the median to the side  $c$  is  $5$  cm long. Find the length of the side  $c$  and the area of the triangle. Draw a sketch first.

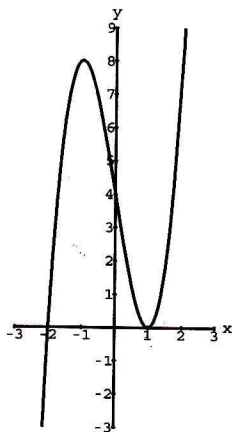
[8 points]

5. Write in the form  $a + bi$ ,  $a, b \in \mathbb{R}$ , the following complex number:

$$(5 - 10i)^2 \cdot (2 + i)^{-1}.$$

[6 points]

6. A graph of the cubic polynomial function is given in the coordinate system. Find the equation of the polynomial.

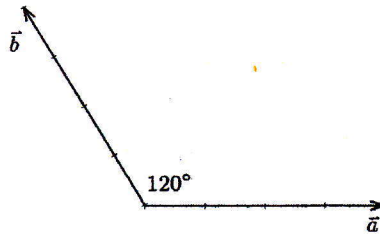


[6 points]

7. An ellipse with its centre in the origin of the coordinate system and vertices  $V_1(2, 0)$  and  $V_2(-2, 0)$  passes through the point  $A\left(\sqrt{3}, \frac{\sqrt{2}}{2}\right)$ . Find the equation of the ellipse and the coordinates of its covertices.

[7 points]

8. Vectors  $\vec{a}$  and  $\vec{b}$  in the picture below are both 4 units long, the angle between them is  $120^\circ$ .



Draw the vector  $\vec{c} = -2\vec{a} + \frac{1}{2}\vec{b}$  and find the scalar products  $\vec{a} \cdot \vec{b}$  and  $\vec{a} \cdot \vec{c}$ .

[8 points]

9. Given  $f(x) = a \cdot 3^{x-1} + b$ ,  $a, b \in \mathbb{R}$ , find such  $a$  and  $b$  that  $f(1) = -1$  and  $f(3) = -17$ . Find the domain  $\mathcal{D}_f$  and the range  $\mathcal{R}_f$  for function  $f$ .

[7 points]

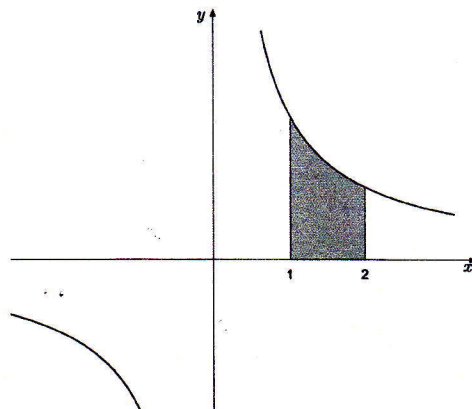
10. Find the zeros of each of the functions  $f(x) = \sin \frac{x}{3}$  and  $g(x) = 2 \cdot \sin \frac{x}{3} + 1$ .

[7 points]

11. Marjetica has 21 girlfriends and 11 boyfriends. Only one of her boyfriends is called Andrej and only one is called Borut. Marjetica is going to invite to the party 3 of her girlfriends and 4 of her boyfriends. In how many ways can she do that? If Marjetica chooses the invitees randomly, what is the probability that Andrej and Borut are both invited?

[6 points]

12. The graph of a function  $f(x) = \frac{a}{x}$  is given. Find such  $a$  that the area of the shaded region in the picture below will be equal to 4.



[7 points]

### HIGHER LEVEL

1. Functions  $f(x) = \frac{2x^2-8}{2x^2+1}$  and  $g(x) = \frac{x}{2} + 1$  are given.

a) Find the inverse  $g^{-1}$  of the function  $g$ .

[2 points]

b) Let  $h(x) = (g \circ f)(x) = g(f(x))$ . Show that  $h(x) = \frac{3x^2-3}{2x^2+1}$ .

[2 points]

c) Find the zeros of the function  $h$ , its extrema and the equation of its horizontal asymptote. Graph the function.

[5 points]

d) Find all real values of  $x$  for which  $2g(x) - 3 \leq h(x)$ .

[6 points]

2. The  $n$ th term of a sequence is  $a_n = \sqrt{n^2 + 4n} - n$ ,  $n \in \mathbb{N}$ .

a) Show with calculation that 1.97 is not a term of the given sequence.

[3 points]

b) Find the limit of the sequence.

[4 points]

c) Let  $\epsilon = \frac{1}{10}$ . Which terms of the sequence are not in the  $\epsilon$ -neighbourhood of 2? Write down your answer.

[4 points]

3. Circle  $K$  has its centre in the point  $S(5, 10)$  and touches the ordinate axis ( $y$ -axis). Line  $y = 7x$  intersects circle  $K$  in points  $A$  and  $B$ .

a) Find the equation of the circle  $K$  and the coordinates of the points  $A$  and  $B$ .

[5 points]

b) Show that  $\angle ASB$  is a right angle.

[3 points]

c) Two tangents are drawn from the origin of the coordinate system to the circle  $K$ . One of these tangents is the ordinate axis. Find the equation of the other tangent and the coordinates of the point where the tangent touches the circle  $K$ .

[6 points]