Interdisciplinary teaching involving geometry

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Fermat meets Pythagoras

1. starting point in schools
2. structure of teaching material
3. discussion
starting point

Physics lessons

• need mathematics as tool
• need mathematics, which is not covered in math lessons at the same time (most of the time)

Math lessons

• Physical contents as means to an end
• No confirmation by an experiment
• No physical interpretations of solutions
requirements before teaching this interdisciplinary lesson

Mathematics:
• Pythagoras’ theorem is known to students
• Functions can be drawn by graphing calculator or computer

Physics:
• Geometrical optics
• Reflection and refraction taught as independent phenomena
background knowledge for “Non-Physicians“

Fermat‘s principle:
the path taken between two points by a ray of light is the path that can be traversed in the least time.
structure of teaching material

- Lesson 1 & 2: Lifeguard problem (ws 1)
  comprehension of problem and solving by graphing calculator resp. Computer

- Lesson 3+: Fermat‘s principle (ws 2)
  Principle will be introduced by lifeguard problem

- further possibilities
  - extreme value tasks
  - bringing together reflection and refraction
Lifeguard problem
Students shall calculate distances with given values
  - two cases
    - Repetition of Pythagoras‘ theorem
    - Shortest way in between
  - Comparing results
  - Students choose own way and compare with each other
  - Modeling a function describing distance between lifeguard and drowning person
  - Minimizing function by using graphing calculator
worksheet 2 (physics)

• Introducing Fermat’s principle

„Light acts like a perfect lifeguard“
worksheet 2 (physics)

- “Snail wants light in its house”
- Transferring figure to coordinate system with given values
- Choosing random point Q on surface and modeling a function describing the ray of light
- Determining shortest distance
- Considering different cases of Q
Solution

\[ t(x) = \frac{\sqrt{26^2 + x^2}}{30.000.000.000} + \frac{\sqrt{11^2 + (50-x)^2}}{22.500.000.000} \]
worksheet 2 (physics)
Extension (Sec II)

Determine general solution analytically

\[ t(x) = \frac{1}{v_1} \sqrt{a^2 + x^2} + \frac{1}{v_2} \sqrt{b^2 + (c - x)^2} \]

\[ t'(x) = 0 \]

\[ \Rightarrow \frac{1}{v_1} x(a^2 + x^2)^{-0.5} + \frac{1}{v_2} (c - x)(b^2 + (c - x)^2)^{-0.5} = 0 \]

\[ \Leftrightarrow v_2 = \frac{(c - x)(b^2 + (c - x)^2)^{-0.5}}{1 - \frac{x(a^2 + x^2)^{-0.5}}{v_1}} \]
further discussion after worksheets

**Mathematics:**
- Discussing procedure in structured way
- Transferring it to other extreme value tasks

**Physics:**
- Generalizing very calculation for any phenomena involving refraction
advantages

• strong interdisciplinary lesson, both subjects are equal
• Mathematical solution will be confirmed by a physical experiment.
• Students get to know “complex” functions and recognize how to handle these by using tools.
• Solution will be achieved by looking at graphs and not by using terms.
  → emphasis on terms can be reduced
interdisciplinarity

- both subjects support each other
- students can see common approaches of both subjects
- When going beyond lesson different views/intentions of both subjects can be experienced.
More examples involving geometry
One classical example
Using geometry when evaluating Elementary analysis

Aim: determine chemical formula of unknown hydrocarbon
Elementary analysis

Steps:
1. Calculate masses and molar masses of carbon and hydrogen of given values
2. Determine mass of oxygen by calculating difference of Oxygen measured in experiment and Oxygen added
3. Divide all calculated molar masses by smallest molar mass determined. Determine least common denominator.
4. Round to natural numbers
Why not using geometry in step 3 and 4?
realization in math class

elementary analysis and theorem of intersecting lines

Discussion